Instructions for Use of Algebra Study Materials

The study materials include four (4) files. The file named “Part A Problems” includes thirty-five (35) questions that are at the beginning algebra level. The file named “Part A Solutions” includes detailed solutions to these problems. The file named “Part B Problems” includes fifty (50) questions that are at the intermediate algebra level. The file named “Part B Solutions” includes detailed solutions to these problems.

These files can be used as preparation for the math placement test or as a general refresher before moving on to college level mathematics courses.

It is recommended that students print the questions, “Part A Problems” and “Part B Problems” but not the solutions. Try each problem, writing every step out in detail. Look at the electronic version of the solution only after attempting the problem. Reading through the detailed solutions may give you the false impression that you know how to do the problem when in fact you do not. Remember that as an instructor does a problem in class, the steps he is taking usually make perfect sense. It is later, when you are attempting to do similar problems yourself, that questions arise. Specific questions about the content of these documents should be directed to the Science/Mathematics Division.

To study precalculus level material please visit the University of California Davis precalculus website: http://www.math.ucdavis.edu/~marx/precalculus.html

Part A Problems

1. Solve the following equation: $5+x=12$

2. Solve the following equation: $6x=36$

3. Solve the following equation: $3=\dfrac{z}{-5}$

4. Evaluate the following expression. $\dfrac{(3+5)^2+|-2|}{2(5-2^3)}$

5. Given that $x=4$, $y=3$, and $z=5$, evaluate the following expression:

   $$(-z)^2+4x-7y$$

6. Solve the following equation: $3(x-1)+7x=-5$

7. Solve the following formula for the variable r: $A=P(1+rt)$

8. Solve the following equation: $x+\dfrac{x}{7}=\dfrac{x}{3}+\dfrac{4}{3}$

9. Ernie has saved \$1,202.50 so far. He is saving to buy a new stereo system and has figured that he has saved 65% of the selling price of the system. What is the selling price of the stereo system Ernie wants to buy?

10. A recent rental bill for a car rental was \$235, including a \$100 rental fee and the rest for mileage at \$0.25 per mile. How many miles did the renter drive the car?

11. Solve the following system of equations by graphing: $\begin{cases} -x+3y=-11 \\ 3x-y=17 \end{cases}$

12. Solve the following system of equations by substitution: $\begin{cases} x-11=3y \\ 6x=2y+34 \end{cases}$

13. Solve the following system of equations by addition (elimination):

$$\begin{cases} \frac{1}{3}x+y=7 \\ \frac{2}{3}x-y=-4 \end{cases}$$

14. Ice cream cones sell for \$1.10 and sundaes sell for \$3.25 at one store. If the receipts for a total of 115 cones and sundaes were \$216.80, how many of each were sold?

15. Add the following polynomials: $\left(2x^2-3x+1\right)+\left(x^2+2x-5\right)$

16. Subtract the following polynomials: $\left(t^3+2t-4\right)-\left(2t^2-t+2\right)$

17. Multiply the following monomials: $(5x)\ \left(7x^3\right)$

18. Multiply the following polynomials. $(7x)\ \left(3x^2+x+5\right)$

19. Multiply the following polynomials. $(2x-4)\ \left(4x^2-3x+2\right)$

20. Multiply the following polynomials. $(x+4)\ (2x-7)$

21. Factor out the greatest common factor. $24a^2b^3+8ab^2$

22. Factor by grouping. $6y^2 + 2y + 9y + 3$

23. Factor completely. $x^2 + 6x - 7$

24. Factor completely. $4x^2 + 8x + 3$

25. Factor completely. $12x^3 + 38x^2 + 6x$

26. Factor completely. $a^2 - 36$

27. Factor completely. $x^3 - 125$

28. Solve the following quadratic equation by factoring. $2x^2 - x - 6 = 0$

29. Use the quadratic formula to solve the following quadratic equation.

$$x^2 + 5x + 6 = 0$$

30. Graph the following quadratic function labeling the vertex, y-intercept, x-intercepts, and one other point: $y = x^2 + 2x - 3$

31. Solve the following inequality expressing the solution set in set builder notation.

$$7 - 5x \le 42$$

32. Solve the following inequality expressing the solution set in set builder notation.

$$8x + 6(2x - 7) \ge 2x - 6$$

33. Write the equation of the line passing through the point $(1,\ -6)$ and parallel to the line $y = \frac{1}{2}x - 8$.

34. Write the equation of the line with slope 3 and passing through the point $(3,\ 2)$.

35. Find the slope of a line perpendicular to the line $y = \frac{9}{17}x + 27$.

Part A Solutions

Problem #1

$5+x=12$ First subtract 5 from both sides of the eqution.

$5+x-5=12-5$ Now combine like terms.

$x=7$

Problem #2

$6x=36$ First divide both sides of the equation by 6.

$\frac{6x}{6}=\frac{36}{6}$ Next, simplify.

$x=6$

Problem #3

$3=\frac{z}{-5}$ First multiply both sides of the equation by -5.

$3(-5)=\frac{z}{-5}(-5)$ Next simplfy.

$-15=\frac{z}{-\cancel{5}}(-\cancel{5})$

$-15=z$

Problem #4

$$\frac{(3+5)^2+|-2|}{2(5-2^3)}$$

Begin simplifying the numerator by doing the addition within the parentheses.

$$\frac{(8)^2+|-2|}{2(5-2^3)}$$

Continue by applying the exponent.

$$\frac{64+|-2|}{2(5-2^3)}$$

Next evaluate the absolute value.

$$\frac{64+2}{2(5-2^3)}$$

Now do the addition in the numerator.

$$\frac{66}{2(5-2^3)}$$

Next simplfy the denominator, beginning within the parentheses.

$$\frac{66}{2(5-8)}$$

Now do the subtraction in the denominator.

$$\frac{66}{2(-3)}$$

Do the multiplication in the denominator.

$$\frac{66}{-6}$$

Now do the division.

$$-11$$

Problem #5

Since $x = 4$, $y = 3$, and $z = 5$:

$(-z)^2 + 4x - 7y$ First substitute the given values for $x, y,$ and z.

$(-5)^2 + 4(4) - 7(3)$ Now apply the order of operations.

$25 + 4(4) - 7(3)$ No parentheses so calculate the exponent first.

$25 + 16 - 7(3)$ Next do multiplication and division from left to right.

$25 + 16 - 21$

$41 - 21$ Now do addition and subtraction from left to right.

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Problem #6

$3(x-1)+7x=-5$ First expand the term to the left (multiply $x-1$ by 3).

$3x-3+7x=-5$ Now combine like terms on the left side of the equation ($3x+7x=10x$).

$10x-3=-5$ Next isolate the variable term by adding 3 to both sides of the equation.

$10x-3+3=-5+3$

$10x=-2$ Now isolate the variable x by multiplying both sides of the equaqtion by the reciprocal of the coefficient of x.

$$10x\left(\frac{1}{10}\right)=-2\left(\frac{1}{10}\right)$$

$$\frac{10}{10}x=\frac{-2}{10}$$

$$x=-\frac{1}{5}$$

Problem #7

$A = P(1+rt)$ We must isolate r on one side of the equation. Begin by dividing both sides of the equation by P.

$$\frac{A}{P} = \frac{\cancel{P}(1+rt)}{\cancel{P}}$$

$\frac{A}{P} = 1+rt$ Now subtract 1 from each side of the equation.

$$\frac{A}{P} - 1 = \cancel{1} + rt - \cancel{1}$$

$\frac{A}{P} - 1 = rt$ Next multiply both sides of the equation by $\frac{1}{t}$ (or divide by t).

$$\left(\frac{A}{P} - 1\right)\frac{1}{t} = (r\cancel{t})\frac{1}{\cancel{t}}$$

$$\left(\frac{A}{P} - 1\right)\frac{1}{t} = r$$

Problem #8

$x+\frac{x}{7}=\frac{x}{3}+\frac{4}{3}$ Eliminate the fractions by multiplying both sides of the equation by 21, which is the least common multiple of 3 and 7.

$$21\left(x+\frac{x}{7}\right)=21\left(\frac{x}{3}+\frac{4}{3}\right)$$

$$21x+\frac{21x}{7}=\frac{21x}{3}+\frac{21(4)}{3}$$

$$21x+\frac{21}{7}x=\frac{21}{3}x+\frac{21}{3}(4)$$

$21x+3x=7x+7(4)$

$24x=7x+28$ Now subtract $7x$ from both sides of the equation.

$24x-7x=\cancel{7x}+28-\cancel{7x}$

$17x=28$ Next multiply both sides of the equation by $\frac{1}{17}$ (or divide by 17).

$$\frac{\cancel{17}x}{\cancel{17}}=\frac{28}{17}$$

$$x=\frac{28}{17}$$

Problem #9

Ernie has saved $1,202.50 so far. He is saving to buy a new stereo system and has figured that he has saved 65% of the selling price of the system. What is the selling price of the stereo system Ernie wants to buy?

Let P = the selling price of the stereo system. Since Ernie has saved 65% of the selling price and he has saved $1,202.50, 65% of P is 1202.5. This translates to the equation:

$65\left(\frac{1}{100}\right)P = 1202.5$ We want to solve for P.

$\frac{65}{100}P = 1202.5$ Multiply both sides of the equation by the reciprocal of the coefficient of P.

$$\left(\frac{\cancel{100}}{\cancel{65}}\right)\frac{\cancel{65}}{\cancel{100}}P = \left(\frac{100}{65}\right)1202.5$$

$P = \frac{120250}{65} = 1850$ So the selling price is $1,850.00

Problem #10

A recent rental bill for a car rental was $235, including a $100 rental fee and the rest for mileage at $0.25 per mile. How many miles did the renter drive the car?

If we let m be the number of miles the renter drove, then $100 + 0.25m$ represents the total cost to rent the car which is $235. So we must solve the equation $100 + 0.25m = 235$.

$100 + 0.25m = 235$ First subtract 100 from each side of the equation.

$100 + 0.25m - 100 = 235 - 100$

$0.25m = 135$ Next divideboth sides of the equation by 0.25

$$\frac{\cancel{0.25}m}{\cancel{0.25}} = \frac{135}{0.25}$$

$m = 540$, So the driver drove 540 miles.

Problem #11

To solve the system by graphing, graph each equation and determine the points of intersection. These points make up the solution set.

$$\begin{cases} -x+3y=-11 \\ 3x-y=17 \end{cases}$$

Since each equation represents a straight line, graph the first equation by finding two points that satisfy the equation. When $x=-1$,

$-x+3y=-11$ becomes $-(-1)+3y=-11$ and solving for y we get;

$1+3y-1=-11-1$

$3y=-12$

$\frac{\cancel{3}y}{\cancel{3}}=\frac{-12}{3}$

$y=-4$ so the point $(-1,\ -4)$ is a point on the first line.

When $x=2$,

$-x+3y=-11$ becomes $-(2)+3y=-11$ and solving for y we get;

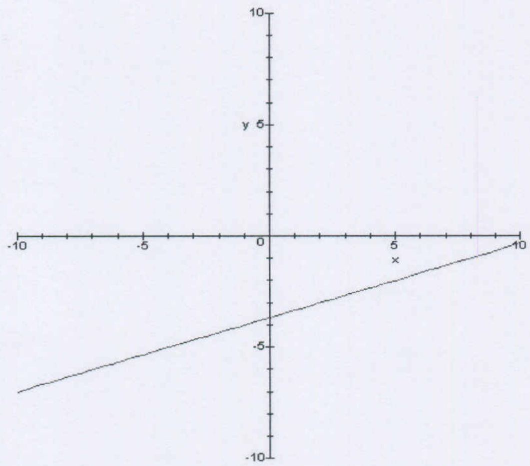
$-2+3y+2=-11+2$

$3y=-9$

$\frac{\cancel{3}y}{\cancel{3}}=\frac{-9}{3}$

$y=-3$ so the point $(2,\ -3)$ is also a point on the first line.

We can now plot these two points to graph the first line.



Now we graph the second equation. When $x = 3$,

$3x - y = 17$ becomes $3(3) - y = 17$ and solving for y we get;

$9 - y - 9 = 17 - 9$

$-y = 8$

$\frac{\cancel{-}y}{\cancel{-1}} = \frac{8}{-1}$

$y = -8$ so the point $(3,\ -8)$ is a point on the second line.

When $x = 7$,

$3x - y = 17$ becomes $3(7) - y = 17$ and solving for y we get;

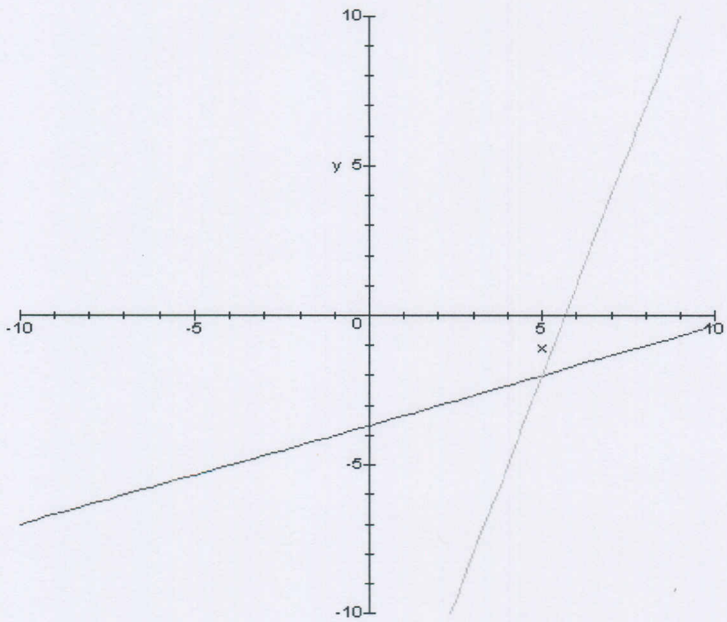
$21 - y - 21 = 17 - 21$

$-y = -4$

$\frac{\cancel{-}y}{\cancel{-1}} = \frac{-4}{-1}$

$y = 4$ so the point $(7,\ 4)$ is also a point on the second line.

We can now plot these two points to graph the second line.



We can now estimate the point of intersection. It appears to be about (5, – 2). If we check this proposed solution in each of the original equations we will confirm that it satisfies both.

Problem #12

$$\begin{cases} x-11=3y \\ 6x=2y+34 \end{cases}$$

To solve by substitution, we begin by solving one equation for one of the variables. Let's pick the first equation $x-11=3y$ and solve for x.

$x\cancel{-11}\cancel{+11}=3y+11$

$x=3y+11$ Now substitute this expression for x into the other equation $6x=2y+34$ and solve for y.

$6(3y+11)=2y+34$

$18y+66=2y+34$

$18y\cancel{+66}\cancel{-66}=2y+34-66$

$18y=2y-32$

$18y-2y=\cancel{2y}-32\cancel{-2y}$

$16y=-32$

$$\frac{\cancel{16}y}{\cancel{16}}=\frac{-32}{16}$$

$y=-2$ Now we can substitute this value for y in either equation and solve for x.

$6x=2(-2)+34$

$6x=-4+34$

$6x=30$

$$\frac{\cancel{6}x}{\cancel{6}}=\frac{30}{6}$$

$x=5$ So the proposed solution is the point $(5,\ -2)$. This proposed solution satisfies all original equations so the solution set is $\{(5,\ -2)\}$

Problem #13

$$\begin{cases} \frac{1}{3}x + y = 7 \\ \frac{2}{3}x - y = -4 \end{cases}$$

To solve the system by addition (elimination), add the left side of the first equation to the left side of the second equation and set that sum equal to the sum of the right sides of each equation. This will eliminate the y variable since they have opposite coefficients.

$$\left(\frac{1}{3}x + y\right) + \left(\frac{2}{3}x - y\right) = (7) + (-4)$$

$$\frac{1}{3}x \cancel{+y} + \frac{2}{3}x \cancel{-y} = 7 - 4$$

$\frac{1}{3}x + \frac{2}{3}x = 3$ Now that the y variable is eliminated, we solve for the x variable.

$$\frac{3}{3}x = 3$$

$x = 3$ Now substitute this value for x in either original equation and solve for y.

$$\frac{1}{3}x + y = 7$$

$$\frac{1}{3}(3) + y = 7$$

$$1 + y = 7$$

$$\cancel{1} + y \cancel{-1} = 7 - 1$$

$y = 6$ So the proposed solution is the point (3, 6). This proposed solution satisfies each original equation so the solution set is $\{(3, 6)\}$.

Problem #14

Ice cream cones sell for $1.10 and sundaes sell for $3.25 at one store. If the receipts for a total of 115 cones and sundaes were $216.80, how many of each were sold?

Let c be the number of cones sold and let s be the number of sundaes sold. The number of cones and sundaes sold is 115 so $c+s=115$. The receipts for these 115 treats was $216.80 so $1.10c+3.25s=216.80$ and we have the following system of equations:

$$\begin{cases} c+s=115 \\ 1.10c+3.25s=216.80 \end{cases}$$

We need one of the variables to have opposite coefficients so we can eliminate it. We can get this by multiplying both sides of the first equation by -1.1 to get:

$$\begin{cases} -1.1c-1.1s=115(-1.1) \\ 1.10c+3.25s=216.80 \end{cases}$$

$$\begin{cases} -1.1c-1.1s=-126.5 \\ 1.1c+3.25s=216.8 \end{cases}$$

$\cancel{-1.1c}-1.1s\cancel{+1.1c}+3.25s=-126.5+216.8$

$-1.1s+3.25s=90.3$

$2.15s=90.3$

$\frac{\cancel{2.15}s}{\cancel{2.15}}=\frac{90.3}{2.15}$

$s=42$ So 42 sundaes were sold. Now substitute this value for s into either original equation and solve for c.

$c+s=115$

$c+(42)=115$

$c\cancel{+42}\cancel{-42}=115-42$

$c=73$ So 73 cones were sold. The proposed solution, $c = 73$ and $s = 42$ satisfies both equations and checks in the wording of the problem so it is the solution.

Problem #15

$\left(2x^2-3x+1\right)+\left(x^2+2x-5\right)$ Write the first polynomial without parentheses.

$2x^2-3x+1+\left(x^2+2x-5\right)$ Next add each term of the second polynomial to the first.

$2x^2-3x+1+x^2+2x+(-5)$

$2x^2-3x+1+x^2+2x-5$ Now combine the like terms.

$\left(2x^2+x^2\right)+(-3x+2x)+(1-5)$

$3x^2+(-x)+(-4)$

$3x^2-x-4$

Problem #16

$\left(t^3+2t-4\right)-\left(2t^2-t+2\right)$ Write the first polynomial without parentheses.

$t^3+2t-4-\left(2t^2-t+2\right)$ Next subtract each term of the second polynomial from the first.

$t^3+2t-4-2t^2-(-t)-2$

$t^3+2t-4-2t^2+t-2$ Now combine like terms.

t^3-2t^2+3t-6

Problem #17

$(5x)\ \left(7x^3\right)$ To multiply monomials, we multiply the numeric coefficients first.

$(5)(x)(7)\left(x^3\right)$

$(5)(7)(x)\left(x^3\right)$

$35(x)\left(x^3\right)$ Then we multiply the variable parts by adding the exponents.

$35\left(x^1\right)\left(x^3\right)$

$35x^{1+3}$

$35x^4$

Problem #18

$(7x)\ \left(3x^2+x+5\right)$ To multiply polynomials, we multiply each term of the second polynomial by each term of the first polynomial.

$(7x)\left(3x^2\right)+(7x)(x)+(7x)(5)$

$21x^3+7x^2+35x$

Problem #19

$(2x-4)\ \left(4x^2-3x+2\right)$ Multiply each term of the second polynomial by each term of the first.

$(2x)\left(4x^2\right)+(2x)(-3x)+(2x)(2)+(-4)\left(4x^2\right)+(-4)(-3x)+(-4)(2)$

$8x^3+\left(-6x^2\right)+4x+\left(-16x^2\right)+12x+(-8)$ Now combine like terms.

$8x^3+\left(-6x^2-16x^2\right)+(4x+12x)-8$

$8x^3-22x^2+16x-8$

Problem #20

$(x+4)\ (2x-7)$ Multiply each term of the second polynomial by each term of the first.

$(x)(2x)+(x)(-7)+(4)(2x)+(4)(-7)$

$2x^2+(-7x)+8x+(-28)$ Now combine like terms.

$2x^2+(-7x+8x)-28$

$2x^2+x-28$

Problem #21

$24a^2b^3+8ab^2$ Note that 8 is the greatest common factor of the numeric coefficients 24 and 8, a is the greatest common factor of a^2 and a, and b^2 is the greatest common factor of b^3 and b^2 so $8ab^2$ is the greatest common factor of $24a^2b^3$ and $8ab^2$.

$$\underbrace{\left(8ab^2\right)(3ab)}_{24a^2b^3}+\underbrace{\left(8ab^2\right)(1)}_{8ab^2}$$

$\left(8ab^2\right)(3ab+1)$

Problem #22

$6y^2+2y+9y+3$ First group terms with common factors.

$(6y^2+2y)+(9y+3)$ Next factor out each groups' greatest common factor.

$(2y)(3y+1)+(3)(3y+1)$ Now factor out the common factor $(3y+1)$ of the addends.

$(3y+1)(2y+3)$

Problem #23

x^2+6x-7 This is a trinomial of the form ax^2+bx+c where $a=1$ so we must find two numbers m and n such that $m+n=b$ (6 in this problem) and $mn=c$ (-7 here). The factorization will then be $(x+m)(x+n)$.

Since the product of the two numbers m and n is negative $(mn=-7)$, one of these numbers must be negative and the other must be positive. Since the sum $m+n=6$ which is positive, the number with the greatest absolute value must be positive. Note that $(7)(-1)=-7$ and $(7)+(-1)=6$ so we let $m=7$ and $n=-1$, giving the factorization $(x+7)(x-1)$.

Problem #24

$4x^2+8x+3$ This is a trinomial of the form ax^2+bx+c where $a\neq 1$ so we must find two numbers m and n such that $m+n=b$ (8 in this problem) and $mn=ac$ (12 here). Then we must use this m and n to split $8x$ into two terms and factor by grouping.

Since the product of the two numbers m and n is positive $(mn=12)$, both of these numbers must be positive or they must both be negative. Since the sum $m+n=8$ which is positive, both numbers must be positive. Note that $(2)(6)=12$ and $(2)+(6)=8$ so we let $m=2$ and $n=6$, and split $8x$ into $2x+6x$. This allows us to factor by grouping.

$$4x^2+\underbrace{8x}_{2x+6x}+3$$

$4x^2+2x+6x+3$

$(4x^2+2x)+(6x+3)$

$2x(2x+1)+3(2x+1)$

$(2x+1)(2x+3)$

Problem #25

$12x^3+38x^2+6x$ First factor out the greatest common factor $2x$.

$2x\left(6x^2+19x+3\right)$ Next factor $\left[6x^2+19x+3\right]$ by grouping.

$2x\left[6x^2+x+18x+3\right]$

$2x\left[\left(6x^2+x\right)+\left(18x+3\right)\right]$

$2x\left[x\left(6x+1\right)+3\left(6x+1\right)\right]$

$2x\left[\left(6x+1\right)\left(x+3\right)\right]$

$2x\left(6x+1\right)\left(x+3\right)$

Problem #26

a^2-36 This is the difference of two squares a^2-6^2 so it can be factored directly from the formula $A^2-B^2=\left(A+B\right)\left(A-B\right)$. Here $A=a$ and $B=6$ so the factorization is $\left(a+6\right)\left(a-6\right)$.

$$\underbrace{a^2-36}_{a^2-6^2}$$

$$\underbrace{a^2-6^2}_{A^2-B^2}=\underbrace{\left(a+6\right)\left(a-6\right)}_{(A+B)(A-B)}$$

Problem #27

x^3-125 This is the difference of two cubes x^3-5^3 so it can be factored directly from the formula $A^3-B^3=\left(A-B\right)\left(A^2+AB+B^2\right)$. Here $A=x$ and $B=5$ so the factorization is $\left(x-5\right)\left(x^2+5x+25\right)$.

$$\underbrace{x^3-125}_{x^3-5^3}$$

$$\underbrace{x^3-5^3}_{A^3-B^3}=\underbrace{\left(x-5\right)\left(x^2+5x+25\right)}_{\left(A^2+AB+B^2\right)}$$

Problem #28

$2x^2 - x - 6 = 0$ First we factor the trinomial on the left side of the equation.

$2x^2 - 4x + 3x - 6 = 0$

$(2x^2 - 4x) + (3x - 6) = 0$

$2x(x-2) + 3(x-2) = 0$

$(x-2)(2x+3) = 0$ Now we apply the zero product principle.

$(x-2) = 0$ or $(2x+3) = 0$

$x - 2 = 0$ or $2x + 3 = 0$

$x - 2 + 2 = 0 + 2$ or $2x + 3 - 3 = 0 - 3$

$x = 2$ or $2x = -3$

$x = 2$ or $\frac{2x}{2} = \frac{-3}{2}$

$x = 2$ or $x = -\frac{3}{2}$

So the solution set is $\left\{2, \ -\frac{3}{2}\right\}$

Problem #29

$x^2 + 5x + 6 = 0$ Here $a = 1$, $b = 5$, and $c = 6$ so applying the quadratic formula we get:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)} = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm \sqrt{1}}{2} = \frac{-5 \pm 1}{2}$$

$x = \frac{-5+1}{2} = \frac{-4}{2} = -2$ or $x = \frac{-5-1}{2} = \frac{-6}{2} = -3$

So the solution set is $\{-2, \ -3\}$

Problem #30

$y = x^2 + 2x - 3$ Here $a = 1,\ b = 2,$ and $c = -3$ and the vertex is the point $\left(\frac{-b}{2a},\ f\left(\frac{-b}{2a}\right)\right)$.

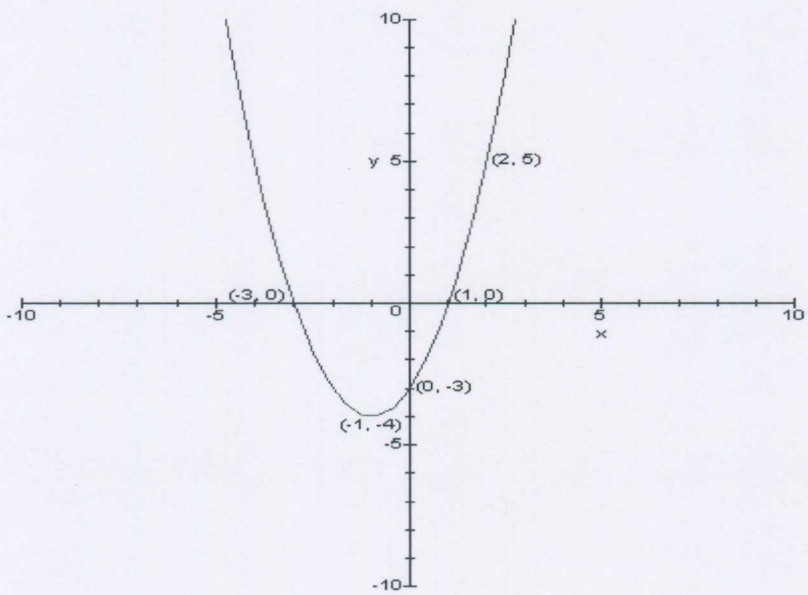
$\frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1$ and $f(-1) = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4$, so the vertex is $(-1,\ -4)$.

The y-intercept occurs at the point where $x = 0$;
$y = 0^2 + 2(0) - 3 = -3$ so the y-intercept is -3 and it occurs at the point $(0,\ -3)$.

The x-intercepts occur at the points where $y = 0$;
$0 = x^2 + 2x - 3 = (x+3)(x-1)$
$(x+3) = 0$ or $(x-1) = 0$
$x = -3$ or $x = 1$
So the x-intercepts occur at the points $(-3,\ 0)$ and $(1,\ 0)$.

To find one other point on the graph we pick a value for x and calculate y;
Let $x = 2$, then $y = 2^2 + 2(2) - 3 = 4 + 4 - 3 = 5$, so the point $(2,\ 5)$ is on the graph.

Now we use this information to plot the graph.



Problem #31

$7-5x \le 42$ We solve the inequality as if it was an equation with the exception that if we multiply or divide both sides by a negative number, we reverse the inequality symbol.

$7-5x-7 \le 42-7$

$-5x \le 35$

$\frac{\cancel{-5}x}{\cancel{-5}} \ge \frac{35}{-5}$ Note that the symbol was reversed here ($\le$ to $\ge$) since we divided by -5.

$x \ge -7$ In set builder notation we write the solution set as $\{x | x \ge -7\}$

Problem #32

$8x+6(2x-7) \ge 2x-6$

$8x+12x-42 \ge 2x-6$

$20x-42 \ge 2x-6$

$20x-42+42 \ge 2x-6+42$

$20x \ge 2x+36$

$20x-2x \ge 2x+36-2x$

$18x \ge 36$

$\frac{\cancel{18}x}{\cancel{18}} \ge \frac{36}{18}$

$x \ge 2$ In set builder notation we write $\{x | x \ge 2\}$.

Problem #33

The line $y=\frac{1}{2}x-8$ has slope $m=\frac{1}{2}$. We see this since the equation is given in slope-intercept form, $y=mx+b$, where m represents the slope of the line and b represents the y-intercept. Since parallel lines have the same slope, the slope of the line we are looking for is also $\frac{1}{2}$. We use this slope and the point $(1, -6)$ in the point slope form for the equation of a line, $(y-y_1)=m(x-x_1)$, with $m=\frac{1}{2}$, $y_1=-6$, and $x_1=1$ to get $(y-(-6))=\frac{1}{2}(x-1)$ or $(y+6)=\frac{1}{2}(x-1)$.

Problem #34

We use the given slope $m = 3$ and the point $(3,\ 2)$ in the point slope form for the equation of a line, $(y - y_1) = m(x - x_1)$, with $m = 3$, $y_1 = 2$, and $x_1 = 3$ to get $(y-2) = 3(x-3)$.

Problem #35

The line $y = \frac{9}{17}x + 27$ has slope $m = \frac{9}{17}$. Since perpendicular lines have negative reciprocal slopes, a line perpendicular to $y = \frac{9}{17}x + 27$ has slope $m = -\frac{17}{9}$.

Part B Problems

1. Solve the following system of equations:

$$\begin{aligned} 2x + y - 2z &= -1 \\ 3x - 3y - z &= 5 \\ x - 2y + 3z &= 6 \end{aligned}$$

2. Solve the following system of equations using matrices and row operations:

$$\begin{aligned} 3x - 4y &= 4 \\ 2x + 2y &= 12 \end{aligned}$$

3. Solve the following system of equations using Cramer's Rule:

$$\begin{aligned} 7x - 11y &= 3 \\ 9x + 2y &= 20 \end{aligned}$$

4. Evaluate the following determinant:

$$\begin{vmatrix} 3 & 5 & 1 \\ 6 & -2 & 2 \\ 8 & -1 & 4 \end{vmatrix}$$

5. Given $f(x) = 2x^2 + x - 5,\ g(x) = x - 4$ find:

   a. $(f + g)(x)$

   b. $(f - g)(x)$

   c. $(fg)(x)$

   d. $\left(\dfrac{f}{g}\right)(x)$

6. Solve the following compound inequality giving the answer in interval notation:

$$2x - 5 \le -11 \quad \text{or} \quad 5x + 1 \ge 6$$

7. Given $f(x) = 2x^3 - 5, \ g(x) = x - 4$ find:

   a. $(f \circ g)(x)$

   b. $(f \circ g)(3)$

   c. $(g \circ f)(x)$

   d. $f^{-1}(x)$

   e. $g^{-1}(x)$

8. Solve the following inequality giving the answer in interval notation:

$$-3(a+2) > 2(a+1)$$

9. Solve the following compound inequality giving the answer in interval notation:

$$2x + 1 > 4x - 3 \quad \text{and} \quad x - 1 \geq 3x + 5$$

10. Solve the equation: $|3x + 2| = 16$

11. Solve the following inequality giving the answer in interval notation:

$$|5x - 2| > 13$$

12. Graph the following inequality: $x - 3y \leq 6$

13. Graph the solution set of the following system of inequalities:

$$4x - 5y \geq -20$$
$$x \geq -3$$

14. Use linear programming to solve the following problem.

You are taking a test that contains computation problems worth 6 points each and word problems worth 10 points each. You can do a computation problem in 2 minutes and a word problem in 4 minutes. You have 40 minutes to take the test and may answer no more than 12 problems. Assuming that you answer every attempted problem correctly, how many of each type of problem must you do to maximize your score and what is the maximum score? (Write the objective function and constraints, graph the constraints labeling all vertices, and solve the problem.)

15. Find the domain of the following function:

$$f(x) = \sqrt{8-2x}$$

16. Simplify the following:

a. $\sqrt[5]{-32(x-2)^5}$

b. $\sqrt[4]{(x+5)^4}$

17. Use rational exponents to simplify the expression and write the result in radical notation. Assume that all variables represent positive real numbers:

$$\sqrt[5]{\sqrt[3]{2x}}$$

18. Simplify, writing the answer with no negative exponents. Assume that all variables represent positive real numbers:

$$\left(2y^{\frac{1}{5}}\right)^4 \div y^{\frac{3}{10}}$$

19. Multiply and simplify. Assume that all variables represent positive real numbers:

$$\sqrt[3]{x-6} \cdot \sqrt[3]{(x-6)^7}$$

20. Perform the indicated operations and simplify:

a. $6\sqrt{7} - \sqrt[3]{x} + 2\sqrt{7} + 5\left(\sqrt[3]{x}\right)$

b. $4\left(\sqrt[3]{x^4 y^2}\right) + 5x\left(\sqrt[3]{xy^2}\right)$

c. $\sqrt[4]{\dfrac{13y^7}{x^{12}}}$

d. $\dfrac{\sqrt{50xy}}{2\sqrt{2}}$

21. Rationalize each denominator:

a. $\dfrac{5}{\sqrt[4]{x}}$

b. $\dfrac{3\sqrt{x} + \sqrt{y}}{\sqrt{y} - 3\sqrt{x}}$

22. Find the solution set to the following equation:

$$\sqrt[4]{x^4 + 4x^2 - 4} = -x$$

23. Find the solution set to the following equation:

$$\sqrt{y+7} + 3 = \sqrt{y+4}$$

24. Divide. Give the answer in $a + bi$ form.

$$\frac{4-3i}{7+2i}$$

25. Complete the square to find the solution set:

$$8x^2 - 10x = 3$$

26. For the points (–3, 5) and (–5, –5)

a. Find the exact distance between the points.
b. Find the midpoint of the line segment joining the points.

27. Use the quadratic formula to find both solutions in the solution set of the following equation. Give complex solutions in $a+bi$ form.

$$-7x = x^2 - 4$$

28. Use the quadratic formula to find both solutions in the solution set of the following equation. Give complex solutions in $a+bi$ form.

$$2x^2 - 5x + 4 = 0$$

29. Given the following quadratic function:

$$f(x) = (x-1)^2 - 4$$

   a. Find the coordinates of the vertex.
   b. Find all x and y intercepts.
   c. Does the function have a maximum or minimum value?
   d. What are the coordinates of the functions minimum or maximum point?

30. Find the solution set for the following equation:

$$x^4 - 68x^2 + 256 = 0$$

31. Solve the following inequality:

$$567 > 7x^2$$

32. Solve the following inequality:

$$\frac{2}{x+5} \geq \frac{1}{x}$$

33. a. Convert 56° to radians. b. Convert $\frac{7\pi}{12}$ radians to degrees.

34. Given a right triangle with hypotenuse measuring 13 units and legs measuring 5 and 12 units. If θ is the angle formed by the hypotenuse and the 12 unit leg, find:

$$\sin(\theta),\ \cos(\theta),\ \tan(\theta),\ \sec(\theta),\ \csc(\theta),\ \cot(\theta)$$

35. In triangle *ABC,* angle *A* measures 60 degrees and angle *B* measures 35 degrees. Find the measure of angle *C*.

36. In triangle *ABC,* angle *A* measures 44 degrees, angle *B* measures 30 degrees, and side *a* measures 7 units. Use the law of sines to find the measure of side *b* to the nearest thousandth of a unit. Note: side *a* is located opposite angle *A*, side *b* is located opposite angle *B*, and side *c* is located opposite angle *C*.

37. In triangle *ABC,* angle *A* measures 75 degrees, side *b* measures 6 units, and side *c* measures 12 units. Use the law of cosines to find the measure of side *a* to the nearest hundredth of a unit. Note: side *a* is located opposite angle *A*, side *b* is located opposite angle *B*, and side *c* is located opposite angle *C*.

38. Rewrite the following expressions in logarithmic form (if given in exponential form) or in exponential form (if given in logarithmic form):

    a. $\log_8 y = 10$

    b. $9^y = 42$

39. Graph the following function, labeling at least three points:

$$f(x) = 7^x$$

40. What is the balance in your account after six years if:

    a. You invest $4,000.00 at 7% compounded monthly?
    b. You invest $4,000.00 at 7% compounded continuously?

41. Write the following as a single logarithm:

$$2\ \log_7 a - 3\ \log_7 b + 5\ \log_7 c$$

42. Graph the following function, labeling at least three points:

$$f(x) = \log_7 x$$

43. Solve for *x* rounding to four decimal places if necessary:

$$4^{x+3} = 1024$$

44. Solve for *x* rounding to four decimal places if necessary:

$$8^{6x^2-7x} = 32768$$

45. Solve for *x* rounding to four decimal places if necessary:

$$\ln 23x = \ln 69$$

46. Solve for x rounding to four decimal places if necessary:

$$\log_9(x+5)-\log_9(2x+9)=0$$

47. Solve for a rounding to four decimal places if necessary:

$$4\ \log_3 a+5\ \log_3 a=6\ \log_3 a+3$$

48. For each equation below:

   a. Identify the conic section.
   b. Write the equation in standard form.
   c. Give the coordinates of all vertices and centers.
   d. Graph the conic section labeling all vertices, centers, and two additional points.

$$4x^2+9y^2-16x-18y=11$$
$$y^2-4x^2-4y-8x=4$$

49. Solve the following system of equations:

$$3x^2+2y^2=36$$
$$4x^2-y^2=4$$

50. Solve the following system of equations:

$$\frac{1}{x}+\frac{3}{y}=4$$

$$\frac{2}{x}-\frac{1}{y}=7$$

Solutions to Part B Problems

Problem #1:

$\boxed{1}$ $2x+y-2z=-1$

$\boxed{2}$ $3x-3y-z=5$

$\boxed{3}$ $x-2y+3z=6$

Start by eliminating the z variable by adding $-2\times\boxed{2}$ to $\boxed{1}$ to get equation $\boxed{A}$:

$\boxed{1}$: $2x+y-2z=-1$

$-2\times\boxed{2}$: $\underline{-6x+6y+2z=-10}$

$\boxed{A}$: $-4x+7y=-11$

Now eliminate the z variable by adding $3\times\boxed{2}$ to $\boxed{3}$ to get equation $\boxed{B}$:

$\boxed{3}$: $x-2y+3z=6$

$3\times\boxed{2}$: $\underline{9x-9y-3z=15}$

$\boxed{B}$: $10x-11y=21$

Next solve the system of two equations $\boxed{A}$ and $\boxed{B}$.

$\boxed{A}$: $-4x+7y=-11$

$\boxed{B}$: $10x-11y=21$ Eliminate y by adding $7\times\boxed{B}$ to $11\times\boxed{A}$.

$11\times\boxed{A}$: $-44x+77y=-121$

$-5\times\boxed{B}$: $\underline{70x-77y=147}$

$26x=26$

$x=1$ Substitute this value for x into $\boxed{B}$ and solve for y.

$10(1)-11y=21,\qquad -11y=11,$

$y=-1$ Now substitute the values of x and y into $\boxed{1}$ and solve for z.

$2(1)+(-1)-2z=-1$

$1-2z=-1$

$-2z=-2$

$z=1$. The proposed solution $(1,\ -1,\ 1)$ checks in all original equations so the solution set is $\{(1,\ -1,\ 1)\}$

Problem #2:

$\begin{bmatrix} 3 & -4 & 4 \\ 2 & 2 & 12 \end{bmatrix}$ This is the augmented matrix that represents the system.

$\begin{bmatrix} 7 & 0 & 28 \\ 2 & 2 & 12 \end{bmatrix}$ This is the result of adding 2 times row 2 to row 1.

$\begin{bmatrix} 1 & 0 & 4 \\ 2 & 2 & 12 \end{bmatrix}$ This is the result of multiplying row 1 by $\frac{1}{7}$.

$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 4 \end{bmatrix}$ This is the result of adding -2 times row 1 to row 2.

$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ This is the result of multiplying row 2 by $\frac{1}{2}$.

The proposed solution (4, 2) checks in both original equations so the solution set is $\{(4, 2)\}$.

Problem #3:

$\begin{bmatrix} 7 & -11 & 3 \\ 9 & 2 & 20 \end{bmatrix}$ This is the augmented matrix that represents the system.

The determinant D= $\begin{vmatrix} 7 & -11 \\ 9 & 2 \end{vmatrix} = (7)\ (2) - (9)\ (-11) = 14 - (-99) = 14 + 99 = 113$

The determinant $D_x = \begin{vmatrix} 3 & -11 \\ 20 & 2 \end{vmatrix} = (3)\ (2) - (20)\ (-11) = 6 - (-220) = 6 + 220 = 226$

The determinant $D_y = \begin{vmatrix} 7 & 3 \\ 9 & 20 \end{vmatrix} = (7)\ (20) - (9)\ (3) = 140 - 27 = 113$

According to Cramer's Rule, $x = \frac{D_x}{D} = \frac{226}{113} = 2$ and $y = \frac{D_y}{D} = \frac{113}{113} = 1$.

The proposed solution (2, 1) checks in each original equation so the solution set is $\{(2, 1)\}$.

Problem #4:

$$\begin{vmatrix} 3 & 5 & 1 \\ 6 & -2 & 2 \\ 8 & -1 & 4 \end{vmatrix} = 3\begin{vmatrix} -2 & 2 \\ -1 & 4 \end{vmatrix} - 6\begin{vmatrix} 5 & 1 \\ -1 & 4 \end{vmatrix} + 8\begin{vmatrix} 5 & 1 \\ -2 & 2 \end{vmatrix} = 3(-8+2) - 6(20+1) + 8(10+2)$$

$$= 3(-6) - 6(21) + 8(12) = -18 - 126 + 96 = -144 + 96 = -48$$

Problem #5

$$f(x)=2x^2+x-5, \quad g(x)=x-4$$

a. $(f+g)(x)=f(x)+g(x)=(2x^2+x-5)+(x-4)=2x^2+x-5+x-4=2x^2+2x-9$

b. $(f-g)(x)=f(x)-g(x)=(2x^2+x-5)-(x-4)=2x^2+x-5-x+4=2x^2-1$

c. $(fg)(x)=f(x)g(x)=(2x^2+x-5)(x-4)=2x^3-8x^2+x^2-4x-5x+20=2x^3-7x^2-9x+20$

d. $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{2x^2+x-5}{x-4}$

Problem #6

First solve the first inequality:

$2x-5\le -11$

$2x-5+5\le -11+5$

$2x\le -6$

$\frac{2x}{2}\le\frac{-6}{2}$

$x\le -3$

Then solve the second inequality:

$5x+1\ge 6$

$5x+1-1\ge 6-1$

$5x\ge 5$

$\frac{5x}{5}\ge\frac{5}{5}$

$x\ge 1$

The original problem was $2x-5\le -11$ or $5x+1\ge 6$ so the solution is $x\le -3$ or $x\ge 1$ which is the union of the two intervals. In interval notation the solution set is $(-\infty,\ -3]\cup[1,\infty)$.

Problem #7

$$f(x)=2x^3-5, \quad g(x)=x-4$$

a. $(f\circ g)(x)=f(g(x))=f(x-4)=2(x-4)^3-5=2(x^2-8x+16)(x-4)-5$

$=2(x^3-4x^2-8x^2+32x+16x-64)-5=2(x^3-12x^2+48x-64)-5$

$=2x^3-24x^2+96x-128-5=2x^3-24x^2+96x-133$

b. $(f\circ g)(3)=f(g(3))=f(3-4)=f(-1)=2(-1)^3-5=2(-1)-5=-2-5=-7$

c. $(g\circ f)(x)=g(f(x))=g(2x^3-5)=(2x^3-5)-4=2x^3-5-4=2x^3-9$

d. $f(x)=2x^3-5$ first replace $f(x)$ with y.

$y=2x^3-5$ now solve for x.

$2x^3-5=y$

$2x^3=y+5$

$x^3=\dfrac{y+5}{2}$

$\sqrt[3]{x^3}=\sqrt[3]{\dfrac{y+5}{2}}$

$x=\sqrt[3]{\dfrac{y+5}{2}}$ next switch the x and y variables.

$y=\sqrt[3]{\dfrac{x+5}{2}}$ finally replace y with $f^{-1}(x)$.

$f^{-1}(x)=\sqrt[3]{\dfrac{x+5}{2}}$

e. $g(x)=x-4$

$y=x-4$

$x-4=y$

$x=y+4$

$y=x+4$

$g^{-1}(x)=x+4$

Problem #8

$$-3(a+2) > 2(a+1)$$
$$-3a-6 > 2a+2$$
$$-5a-6 > 2$$
$$-5a > 8$$
$$a < -\frac{8}{5}$$
$$\left(-\infty,\ -\frac{8}{5}\right)$$

Problem #9

First solve the first inequality:

$$2x+1 > 4x-3$$
$$2x+1-4x > 4x-3-4x$$
$$-2x+1 > -3$$
$$-2x+1-1 > -3-1$$
$$-2x > -4$$

Then solve the second inequality:

$$x-1 \geq 3x+5$$
$$x-1-3x \geq 3x+5-3x$$
$$-2x-1 \geq 5$$
$$-2x-1+1 \geq 5+1$$
$$-2x \geq 6$$

$\dfrac{-2x}{-2} < \dfrac{-4}{-2}$ ← Note symbol reversal → $\dfrac{-2x}{-2} \leq \dfrac{6}{-2}$

$x < 2$ $\qquad$ $x \leq -3$

The original problem was $2x+1 > 4x-3$ and $x-1 \geq 3x+5$ so the solution is $x < 2$ and $x \leq -3$ which is the intersection of the two intervals. In interval notation the solution set is $(-\infty,\ 2) \cap (-\infty,\ -3]$ which is equivalent to $(-\infty,\ -3]$.

Problem #10

$$|3x+2| = 16$$
$$3x+2 = 16 \quad \text{or} \quad 3x+2 = -16$$
$$3x = 14 \quad \text{or} \quad 3x = -18$$
$$x = \frac{14}{3} \quad \text{or} \quad x = -6$$
$$\left\{-6,\ \frac{14}{3}\right\}$$

Problem #11

$|5x-2|>13$

$5x-2>13 \quad \text{or} \quad -(5x-2)>13$

$5x-2+2>13+2 \quad \text{or} \quad -5x+2-2>13-2$

$5x>15 \quad \text{or} \quad -5x>11$

$$\frac{5x}{5}>\frac{15}{5} \quad \text{or} \quad \frac{-5x}{-5}<\frac{11}{-5}$$

$x>3$ or $x<-\frac{11}{5}$ which in interval notation is $\left(-\infty,\ -\frac{11}{5}\right)\cup(3,\ \infty)$

Problem #12

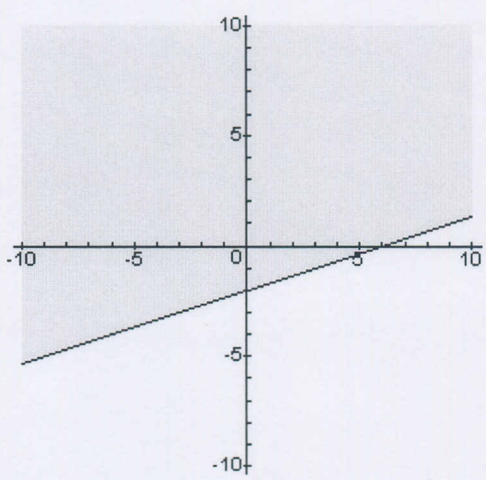
First graph the line $x-3y=6$ as solid line and pick a test point that is not on the line. Consider the test point (0, 0) which lies above the line. We now substitute 0 for x and 0 for y in the original inequality and see if the result is a true statement:

$x-3y\le 6$

$(0)-3(0)\le 6$

$0\le 6$ is a true statement so we shade (yellow in graph below) the side of the line containing the test point.

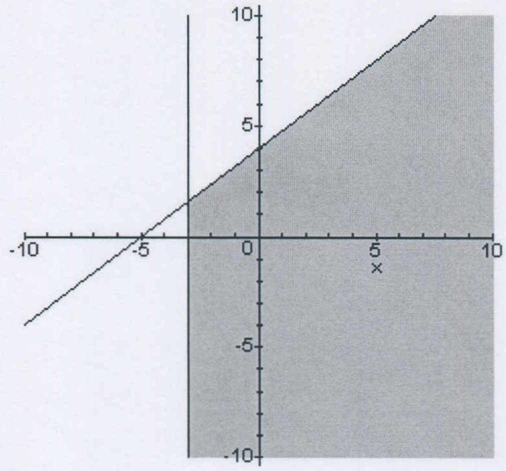
$x-3y\le 6$



Problem #13

Graph both inequalities in the system and shade (green on graph below) the common region.

$$4x - 5y \geq -20$$
$$x \geq -3$$



Problem #14

Let S be the test score, C be the number of computation problems done, and W be the number of word problems done. Since the objective is to maximize the score, the objective function is $S = 6C + 10W$.

The constraints are:

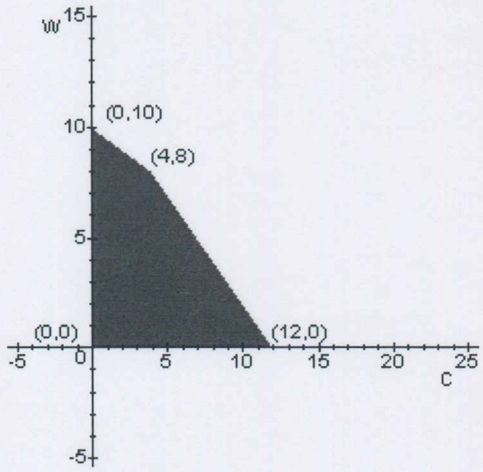
$2C + 4W \leq 40$ (since you have a maximum of 40 minutes to take the test)

$C + W \leq 12$ (since you can answer a maximum of 12 questions)

$C \geq 0$ (since the number of computation problems answered can not be negative)

$W \geq 0$ (since the number of word problems answered can not be negative)

Now we graph the constraints on the same set of coordinate axes labeling the vertices.



To solve the problem we substitute the values of the vertices into the objective function and determine which vertex maximizes the function.

(0, 0) $S = 6(0) + 10(0) = 0$

(0, 10) $S = 6(0) + 10(10) = 0 + 100 = 100$

(4, 8) $S = 6(4) + 10(8) = 24 + 80 = 104$

(12, 0) $S = 6(12) + 10(0) = 72 + 0 = 72$

104 is the highest score and it is obtained by answering 4 computation problems and 8 word problems.

<u>Problem #15</u>

The domain of the function $f(x) = \sqrt{8-2x}$ is the set of all possible values for the x variable. Since the square root of a negative number is not a real number, the radicand, $8-2x$ can not be negative. We must find what values of x make $8-2x \geq 0$.

$$8-2x \geq 0$$
$$8-2x-8 \geq 0-8$$
$$-2x \geq -8$$
$$\frac{-2x}{-2} \leq \frac{-8}{-2}$$
$$x \leq 4$$

Since x must be less than or equal to 4, the domain of the function in set builder notation is $\{x | x \leq 4\}$ and in interval notation is $(-\infty, 4]$.

Problem #16

a. $\sqrt[5]{-32(x-2)^5} = \sqrt[5]{(-2)^5(x-2)^5} = -2(x-2)$ Note: Odd index, absolut value not needed.

b. $\sqrt[4]{(x+5)^4} = |x+5|$ Note: Even index, absolute value needed.

Problem #17

We convert the expression to rational exponent notation, apply the laws of exponents, and then convert back to radical notation:

$$\sqrt[5]{\sqrt[3]{2x}} = \sqrt[5]{(2x)^{\frac{1}{3}}} = \left((2x)^{\frac{1}{3}}\right)^{\frac{1}{5}} = (2x)^{\frac{1}{3}\cdot\frac{1}{5}} = (2x)^{\frac{1}{15}} = \sqrt[15]{2x}$$

Problem #18

$$\left(2y^{\frac{1}{5}}\right)^4 \div y^{\frac{3}{10}} = \frac{\left(2y^{\frac{1}{5}}\right)^4}{y^{\frac{3}{10}}} = \frac{(2)^4\left(y^{\frac{1}{5}}\right)^4}{y^{\frac{3}{10}}} = \frac{16y^{\frac{1}{5}\cdot 4}}{y^{\frac{3}{10}}} = \frac{16y^{\frac{4}{5}}}{y^{\frac{3}{10}}} = 16y^{\frac{4}{5}-\frac{3}{10}} = 16y^{\frac{8}{10}-\frac{3}{10}} = 16y^{\frac{5}{10}} = 16y^{\frac{1}{2}}$$

Problem #19

$$\sqrt[3]{x-6}\cdot\sqrt[3]{(x-6)^7} = \sqrt[3]{(x-6)\cdot(x-6)^7} = \sqrt[3]{(x-6)^{1+7}} = \sqrt[3]{(x-6)^8} = \sqrt[3]{(x-6)^{6+2}} = \sqrt[3]{(x-6)^6\cdot(x-6)^2}$$

$$= \sqrt[3]{\left((x-6)^2\right)^3\cdot(x-6)^2} = (x-6)^2\sqrt[3]{(x-6)^2}$$

Problem #20

a.) $6\sqrt{7} - \sqrt[3]{x} + 2\sqrt{7} + 5\sqrt[3]{x}$ Rewrite with like terms grouped.

$\left(6\sqrt{7} + 2\sqrt{7}\right) + \left(-\sqrt[3]{x} + 5\sqrt[3]{x}\right)$ Combine like terms.

$8\sqrt{7} + 4\sqrt[3]{x}$

b.) $4\sqrt[3]{x^4y^2} + 5x\sqrt[3]{xy^2}$ Rewrite as like terms by factoring radicands.

$4\sqrt[3]{x^3xy^2} + 5x\sqrt[3]{xy^2}$

$4x\sqrt[3]{xy^2} + 5x\sqrt[3]{xy^2}$ Combine like terms.

$9x\sqrt[3]{xy^2}$

c.) $\sqrt[4]{\dfrac{13y^7}{x^{12}}}$ Apply the quotient rule for radicals.

$\dfrac{\sqrt[4]{13y^7}}{\sqrt[4]{x^{12}}}$ Factor radicands.

$\dfrac{\sqrt[4]{13y^4y^3}}{\sqrt[4]{\left(x^3\right)^4}}$

$\dfrac{y\sqrt[4]{13y^3}}{x^3}$

d.) $\dfrac{\sqrt{50xy}}{2\sqrt{2}}$ Factor radicands and remove perfect squares.

$$\frac{\sqrt{(25)(2)xy}}{2\sqrt{2}} = \frac{\sqrt{\left(5^2\right)(2)xy}}{2\sqrt{2}} = \frac{5\sqrt{2xy}}{2\sqrt{2}} = \frac{5\sqrt{2}\sqrt{xy}}{2\sqrt{2}} = \frac{5\sqrt{xy}}{2}$$

Problem #21

a.) $\frac{5}{\sqrt[4]{x}}$ Multiply numerator and denominator by a factor that removes the radical from the denominator.

$\frac{5}{\sqrt[4]{x}}\cdot\frac{\sqrt[4]{x^3}}{\sqrt[4]{x^3}}$ Note that you are multiplying the original expression by 1 since $\frac{\sqrt[4]{x^3}}{\sqrt[4]{x^3}}=1.$

$$\frac{5\sqrt[4]{x^3}}{\sqrt[4]{x\cdot x^3}}=\frac{5\sqrt[4]{x^3}}{\sqrt[4]{x^4}}=\frac{5\sqrt[4]{x^3}}{x}$$

b.) $\frac{3\sqrt{x}+\sqrt{y}}{\sqrt{y}-3\sqrt{x}}$ Multiply numerator and denominator by the conjugate of the denominator.

$\frac{3\sqrt{x}+\sqrt{y}}{\sqrt{y}-3\sqrt{x}}\cdot\frac{\sqrt{y}+3\sqrt{x}}{\sqrt{y}+3\sqrt{x}}$ Note that you are multiplying by 1 since $\frac{\sqrt{y}+3\sqrt{x}}{\sqrt{y}+3\sqrt{x}}=1.$

$$\frac{\left(3\sqrt{x}\right)\left(\sqrt{y}\right)+\left(3\sqrt{x}\right)\left(3\sqrt{x}\right)+\left(\sqrt{y}\right)\left(\sqrt{y}\right)+\left(\sqrt{y}\right)\left(3\sqrt{x}\right)}{\left(\sqrt{y}\right)\left(\sqrt{y}\right)+\left(\sqrt{y}\right)\left(3\sqrt{x}\right)-\left(3\sqrt{x}\right)\left(\sqrt{y}\right)-\left(3\sqrt{x}\right)\left(3\sqrt{x}\right)}$$

$$\frac{\left(3\sqrt{xy}\right)+\left(9\sqrt{x\cdot x}\right)+\left(\sqrt{y\cdot y}\right)+\left(3\sqrt{xy}\right)}{\left(\sqrt{y\cdot y}\right)+\left(3\sqrt{xy}\right)-\left(3\sqrt{xy}\right)-\left(9\sqrt{x\cdot x}\right)}$$

$$\frac{\left(3\sqrt{xy}\right)+\left(9\sqrt{x^2}\right)+\left(\sqrt{y^2}\right)+\left(3\sqrt{xy}\right)}{\left(\sqrt{y^2}\right)-\left(9\sqrt{x^2}\right)}$$

$$\frac{\left(3\sqrt{xy}\right)+9x+y+\left(3\sqrt{xy}\right)}{y-9x}$$

$$\frac{6\sqrt{xy}+9x+y}{y-9x}$$

Problem #22

$$\sqrt[4]{x^4+4x^2-4}=-x$$

$$\left(\sqrt[4]{x^4+4x^2-4}\right)^4=(-x)^4$$

$$x^4+4x^2-4=x^4$$

$$4x^2-4=0$$

$$4x^2=4$$

$$x^2=1$$

$$x=\pm 1$$

$x=1$ or $x=-1$

$x=-1$ checks in the original equation so it is in the solution set.

Check $x=1$;

Left side: $\sqrt[4]{(1)^4+4(1)^2-4}=\sqrt[4]{1}=\pm 1$

Right side: $-(\pm 1)=\pm 1$, but $\pm 1\neq -1$ so $x=1$ is an extraneous root and not in the solution set. The solution set is $\{-1\}$.

Problem #23

$$\sqrt{y+7}+3=\sqrt{y+4}$$

$$\left(\sqrt{y+7}+3\right)^2=\left(\sqrt{y+4}\right)^2$$

$$\left(\sqrt{y+7}+3\right)\left(\sqrt{y+7}+3\right)=y+4$$

$$(y+7)+3\sqrt{y+7}+3\sqrt{y+7}+9=y+4$$

$$y+16+6\sqrt{y+7}=y+4$$

$$16+6\sqrt{y+7}=4$$

$$6\sqrt{y+7}=-12$$

$$\sqrt{y+7}=-2$$

$$\left(\sqrt{y+7}\right)^2=(-2)^2$$

$$y+7=4$$

$y=-3$ Now check in the original equation;
Left side: $\sqrt{(-3)+7}+3=\sqrt{4}+3=\pm 2+3=5$ or 1
Right side: $\sqrt{(-3)+4}=\sqrt{1}=\pm 1=1$ or -1, which not is the same as 5 or 1 so -3 is extraneous, there is no real solution, and the solution set is $\{\ \}$.

Problem #24

$\frac{4-3i}{7+2i}$ First we multiply both numerator and denominator by the conjugate of the denominator.

$$\frac{4-3i}{7+2i}\cdot\frac{7-2i}{7-2i}=\frac{(4)(7)+(4)(-2i)+(-3i)(7)+(-3i)(-2i)}{(7)(7)+(7)(-2i)+(2i)(7)+(2i)(-2i)}=\frac{28-8i-21i+6i^2}{49-14i+14i-4i^2}$$

$$=\frac{28-29i+6(-1)}{49-4(-1)}=\frac{22-29i}{53}=\frac{22}{53}-\frac{29}{53}i$$

Problem #25

$8x^2-10x=3$ First transform the leading coefficient to 1 by dividing both sides by 8.

$$\frac{8x^2}{8}-\frac{10x}{8}=\frac{3}{8}$$

$x^2-\frac{5}{4}x=\frac{3}{8}$ Now add the square of one half the coefficient of x $\left(\frac{1}{2}\cdot\left(-\frac{5}{4}\right)\right)^2=\frac{25}{64}$ to complete the square on the left side.

$x^2-\frac{5}{4}x+\frac{25}{64}=\frac{3}{8}+\frac{25}{64}$ Next write the left hand side as the perfect square.

$\left(x-\frac{5}{8}\right)^2=\frac{49}{64}$ Now we apply the square root property to find the two solutions.

$x-\frac{5}{8}=\sqrt{\frac{49}{64}}$ or $x-\frac{5}{8}=-\sqrt{\frac{49}{64}}$

$x-\frac{5}{8}=\frac{7}{8}$ or $x-\frac{5}{8}=-\frac{7}{8}$

$x=\frac{12}{8}$ or $x=-\frac{2}{8}$

$x=\frac{3}{2}$ or $x=-\frac{1}{4}$ so the solution set is $\left\{-\frac{1}{4},\ \frac{3}{2}\right\}$ since both solutions check in the original equation.

Problem #26

a. $(-3,\ 5),\ (-5,\ -5)$ Use the distance formula.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{[(-5)-(-3)]^2+[(-5)-(5)]^2}=\sqrt{(-2)^2+(-10)^2}$$

$$=\sqrt{4+100}=\sqrt{104}=\sqrt{4\cdot 26}=\sqrt{(2)^2\cdot 26}=2\sqrt{26}$$

b. $(-3,\ 5),\ (-5,\ -5)$ Use the midpoint formula.

$$\text{Midpoint}=\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)=\left(\frac{-3+(-5)}{2},\ \frac{5+(-5)}{2}\right)=\left(\frac{-8}{2},\ \frac{0}{2}\right)=(-4,\ 0)$$

Problem #27

First put the quadratic in general form $ax^2+bx+c=0$ to determine a,b,c.

$-7x=x^2-4$

$x^2+7x-4=0$ so $a=1,\ b=7,\ c=-4,$ and we can use the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

$$x=\frac{-7\pm\sqrt{(7)^2-4(1)(-4)}}{2(1)}=\frac{-7\pm\sqrt{65}}{2}$$ so the solution set is $$\left\{\frac{-7+\sqrt{65}}{2},\ \frac{-7-\sqrt{65}}{2}\right\}$$

Problem #28

The quadratic is in general form $ax^2+bx+c=0$ so we can determine a,b,c.

$2x^2-5x+4=0$ so $a=2,\ b=-5,\ c=4,$ and we can use the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(4)}}{2(2)}=\frac{5\pm\sqrt{-7}}{4}=\frac{5}{4}\pm\frac{\sqrt{7}}{4}i$$ so the solution set is $$\left\{\frac{5}{4}+\frac{\sqrt{7}}{4}i,\ \frac{5}{4}-\frac{\sqrt{7}}{4}i\right\}$$

Problem #29

a. We first put the quadratic in general form.

$$f(x) = (x-1)^2 - 4 = (x-1)(x-1) - 4 = x^2 - x - x + 1 - 4 = x^2 - 2x - 3$$

The coordinates of the vertex are $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ and $a = 1,\ b = -2.$

$\frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$ and $f\left(\frac{-b}{2a}\right) = f(1) = (1)^2 - 2(1) - 3 = -4$ so the coordinates

of the vertex are $(1,\ -4)$.

b. The y-intercept occurs when $x = 0$: $y = x^2 - 2x - 3 = (0)^2 - 2(0) - 3 = -3$ so the y-intercept is -3.

The x-intercepts occur when $y = 0$: $0 = x^2 - 2x - 3$

$$0 = (x+1)(x-3)$$

$$(x+1) = 0 \text{ or } (x-3) = 0$$

$x = -1$ or $x = 3$ so the x-intercepts are -1 and 3.

c. Since a is positive, the parabola which is the graph of this quadratic function opens upward giving the function a minimum value.

d. The minimum point is the vertex so its' coordinates are $(1,\ -4)$.

Problem #30

$x^4 - 68x^2 + 256 = 0$ is quadratic in form since the substitution $u = x^2$ gives the equation;
$u^2 - 68u + 256 = 0$ which is quadratic and can be solved for u by factoring.
$(u-4)(u-64) = 0$
$(u-4) = 0$ or $(u-64) = 0$
$u = 4$ or $u = 64$ but now we must solve for x. Recall that $u = x^2$.
When $u = 4$; $x^2 = 4$ so $x = \pm 2$. When $u = 64$; $x^2 = 64$ so $x = \pm 8$.
So the solution set is $\{-2,\ 2,\ -64,\ 64\}$.

Problem #31

$567 > 7x^2$ is the same as $7x^2 < 567$.
Since $7x^2 < 567$ we begin by dividing each side of the inequality by 7.
$x^2 < 81$ Now find the solutions to $x^2 = 81$.
$x = \pm 9$ Use these numbers to split the real line into intervals:
$(-\infty,\ -9)$, $(-9,\ 9)$, $(9,\ \infty)$ and pick a test point in each interval.
For $(-\infty,\ -9)$ use -10 and test in the original inequality.
$7x^2 < 567$
$7(-10)^2 = 700$ but $700 \not< 567$ so the interval $(-\infty,\ -9)$ is not in the solution set.
For $(-9,\ 9)$ use 0 and test in the original inequality.
$7x^2 < 567$
$7(0)^2 = 0$ and $0 < 567$ so the interval $(-9,\ 9)$ is in the solution set.
For $(9,\ \infty)$ use 10 and test in the original inequality.
$7x^2 < 567$
$7(10)^2 = 700$ but $700 \not< 567$ so the interval $(9,\ \infty)$ is not in the solution set and the solution set is the interval $(-9,\ 9)$.

Problem #32

$$\frac{2}{x+5} \geq \frac{1}{x}$$

$\frac{2}{x+5} - \frac{1}{x} \geq 0$ This inequality is undefined when $x = -5$ or 0 and the solution to $\frac{2}{x+5} = \frac{1}{x}$ is 5. Use these values of x to split the real line into intervals $(-\infty,\ -5)$, $(-5,\ 0)$, $(0,\ 5)$, $(5,\ \infty)$ and pick a test point in each interval.

For $(-\infty,\ -5)$, use -10.

$\frac{2}{(-10)+5} \geq \frac{1}{(-10)}$ gives $-\frac{2}{5} \geq -\frac{1}{10}$ which is false, so the interval $(-\infty,\ -5)$ is not in the solution set.

For $(-5,\ 0)$, use -1.

$\frac{2}{(-1)+5} \geq \frac{1}{(-1)}$ gives $-\frac{1}{2} \geq -1$ which is true, so the interval $(-5,\ 0)$ is in the solution set.

For $(0,\ 5)$, use 1.

$\frac{2}{(1)+5} \geq \frac{1}{(1)}$ gives $\frac{1}{3} \geq 1$ which is false, so the interval $(0,\ 5)$ is not in the solution set.

For $(5,\ \infty)$, use 10.

$\frac{2}{(10)+5} \geq \frac{1}{(10)}$ gives $\frac{2}{15} \geq \frac{1}{10}$ which is true, so the interval $(5,\ \infty)$ is in the solution set.

We must now test the interval endpoints, -5, 0, and 5.

$\frac{2}{(-5)+5} \geq \frac{1}{(-5)}$ gives $\frac{2}{0} \geq -\frac{1}{5}$ but this is undefined so -5 is not in the solution set.

$\frac{2}{(0)+5} \geq \frac{1}{(0)}$ gives $\frac{2}{5} \geq \frac{1}{0}$ but this is undefined so 0 is not in the solution set.

$\frac{2}{(5)+5} \geq \frac{1}{(5)}$ gives $\frac{1}{5} \geq \frac{1}{5}$ and this is true so 5 is in the solution set, and the solution set is,

$(-5,\ 0) \cup [5,\ \infty)$

Problem #33

$56° \cdot \frac{\pi}{180°} = \frac{\cancel{56}\pi}{\cancel{180}} = \frac{14\pi}{45}$ Note: When units are not given, radians are understood.

$\frac{7\pi}{12} \cdot \frac{180°}{\pi} = \frac{7\cancel{\pi}180°}{12\cancel{\pi}} = \frac{1260°}{12} = 105°$

Problem #34

$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{5}{13}, \quad \cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{12}{13}, \quad \tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{5}{12}$

$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{13}{12}, \quad \csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{13}{5}, \quad \cot\theta = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{12}{5}$

Problem #35

The sum of the measures of the three angles of a triangle is 180°. $A + B + C = 180°$.

Since $A = 60°, \quad B = 35°,$

$60° + 35° + C = 180°$

$C = 180° - 60° - 35°$

$C = 85°$

Problem #36

According to the Law of Sines, $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$, so

$$\frac{b}{\sin B}=\frac{a}{\sin A}$$

$$\frac{b}{\sin 30°}=\frac{7}{\sin 44°}$$

$$b=\frac{7\sin 30°}{\sin 44°}\approx 5.038 \text{ units}$$

Problem #37

According to the Law of Cosines: $a^2=b^2+c^2-2bc\cos A$, so

$$a^2=(6)^2+(12)^2-2(6)(12)\cos 75°$$

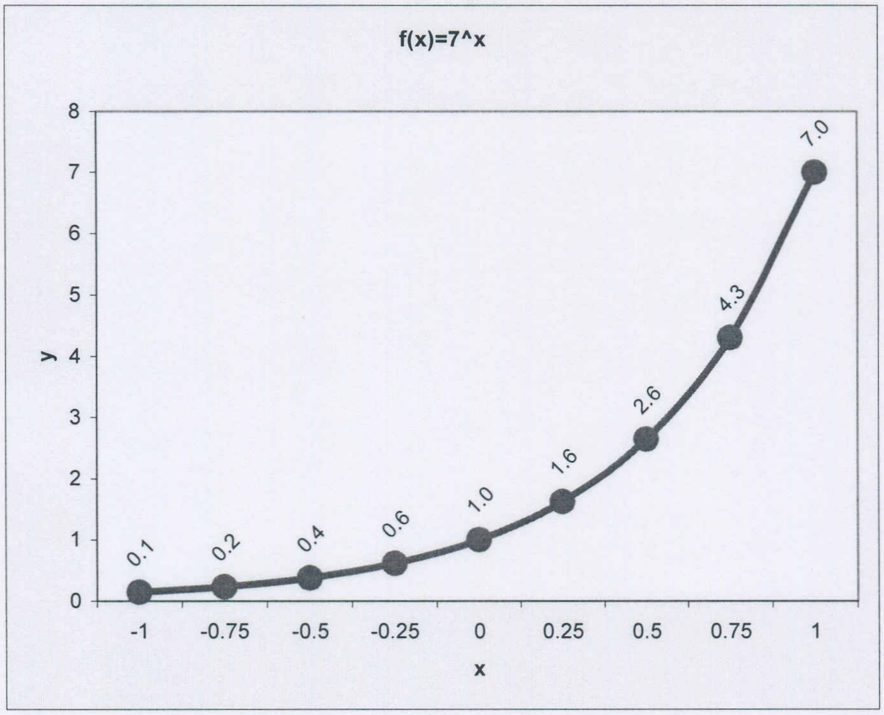
$$a=\sqrt{(6)^2+(12)^2-2(6)(12)\cos 75°}\approx 11.947 \text{ units}$$

Problem #38

a. In exponential form, $\log_8 y=10$ becomes $8^{10}=y$.

b. In logarithmic form, $9^y=42$ becomes $\log_9 42=y$.

Problem #39



Note: The points on the graph are labeled with the value of the function (y-coordinate) only due to limitations with the software used here. Hand-drawn graphs should include both coordinates when labeling points.

Problem #40

a. Compounded monthly:

Use the formula $A = P\left(1+\frac{r}{k}\right)^{kt}$ with $P = \$4,000.00$, $r = 0.07$, $k = 12$, $t = 6$.

$$A = \$4000\left(1+\frac{0.07}{12}\right)^{(12\cdot 6)} \approx \$6080.42$$

Note: Problems involving money should be rounded to the nearest penny.

b. Compounded continuously:

Use the formula $A = Pe^{rt}$ with $P = \$4,000.00$, $r = 0.07$, $t = 6$.

$A = \$4000e^{(0.07\cdot 6)} \approx \$6087.85$ (Rounded to the nearest penny)

Problem #41

Write as a single logarithm: $2\log_7 a - 3\log_7 b + 5\log_7 c$

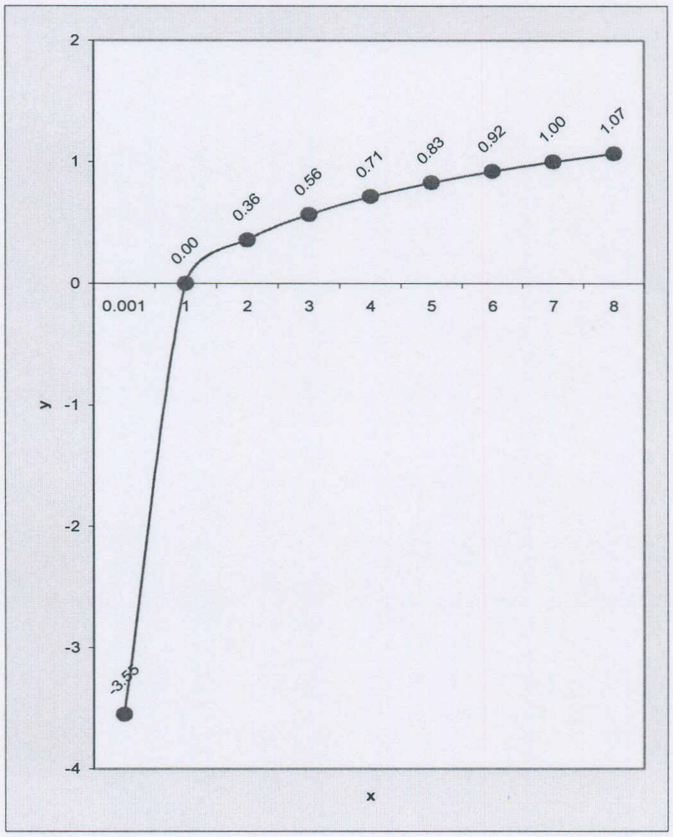
$\log_7 a^2 - \log_7 b^3 + \log_7 c^5$

$\log_7 \dfrac{a^2}{b^3} + \log_7 c^5$

$\log_7 \dfrac{a^2 c^5}{b^3}$

Problem #42

$y = \log_7 x$



Note: The points on the graph are labeled with the value of the function (y-coordinate) only due to limitations with the software used here. Hand-drawn graphs should include both coordinates when labeling points.

Problem #43

$4^{x+3} = 1024$ By taking the logarithm of both sides we can remove the variable from the exponent.

$\ln 4^{x+3} = \ln 1024$

$(x+3)\ln 4 = \ln 1024$

$x+3 = \dfrac{\ln 1024}{\ln 4}$

$x = \dfrac{\ln 1024}{\ln 4} - 3$

$x = 5-3$

$x = 2,$ which checks in the original equation so the solution set is $\{2\}$.

Problem #44

$8^{6x^2-7x} = 32768$

$\ln 8^{6x^2-7x} = \ln 32768$

$\left(6x^2-7x\right)\ln 8 = \ln 32768$

$6x^2-7x = \dfrac{\ln 32768}{\ln 8}$

$6x^2-7x = 5$

$6x^2-7x-5=0$ Now use the quadratic formula with $a=6,\ b=-7,\ c=-5.$

$$x = \frac{-(-7)\pm\sqrt{(-7)^2-4(6)(-5)}}{2(6)} = \frac{7\pm 13}{12} = -\frac{1}{2} \text{ or } \frac{5}{3}.$$

Both of these check in the original equation so the solution set is $\left\{-\dfrac{1}{2},\ \dfrac{5}{3}\right\}$.

Problem #45

$\ln 23x = \ln 69$

$23x = 69$

$x = 3,$ which checks in the original equation so the solution set is $\{3\}$.

Problem #46

$\log_9(x+5)-\log_9(2x+9)=0$

$\log_9(x+5)=\log_9(2x+9)$

$(x+5)=(2x+9)$

$-x=4$

$x=-4,$ which checks in the original equation so the solution set is $\{-4\}$.

Problem #47

$4\log_3 a+5\log_3 a=6\log_3 a+3$

$4\log_3 a+5\log_3 a-6\log_3 a=3$

$\log_3 a^4+\log_3 a^5-\log_3 a^6=3$

$\log_3 a^4a^5-\log_3 a^6=3$

$\log_3 a^9-\log_3 a^6=3$

$\log_3 \frac{a^9}{a^6}=3$

$\log_3 a^3=3$

$3^{\log_3 a^3}=3^3$

$a^3=27$

$(a^3)^{\frac{1}{3}}=(27)^{\frac{1}{3}}$

$a=3,$ which checks in the original equation so the solution set is $\{3\}$.

Problem #48

The first conic section can be identified by putting the equation in standard form.

$4x^2+9y^2-16x-18y=11$ Start by completing the square twice (once for each variable).

$$\left(4x^2-16x\right)+\left(9y^2-18y\right)=11$$

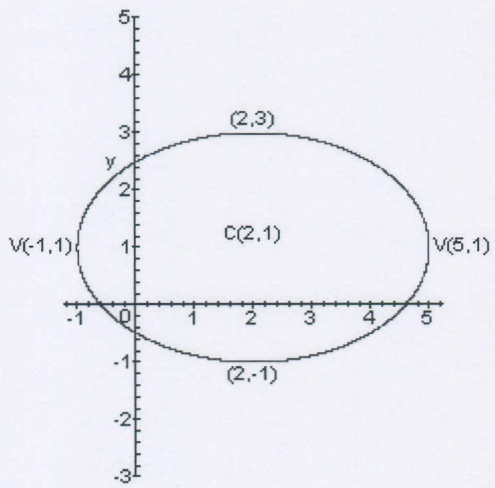
$$4\left(x^2-4x\right)+9\left(y^2-2y\right)=11$$

$$4\left(x^2-4x+4\right)+9\left(y^2-2y+1\right)=11+16+9$$

$4(x-2)^2+9(y-1)^2=36$ Now divide both sides of the equation by 36.

$$\frac{4(x-2)^2}{36}+\frac{9(y-1)^2}{36}=\frac{36}{36}$$

$\dfrac{(x-2)^2}{9}+\dfrac{(y-1)^2}{4}=1$ This is the standard form of an ellipse with center (2, 1), major axis parallel to the x-axis (since 9 is greater than 4), vertices located 3 units to the right and left of the center, (5, 1) and $(-1,\ 1)$ and points along the minor axis located 2 units above and below the center, (2, 3) and $(2,\ -1)$.



The next conic section can be identified by putting the equation in standard form.

$y^2 - 4x^2 - 4y - 8x = 4$ Start by completing the square twice (once for each variable).

$\left(y^2 - 4y\right) - \left(4x^2 + 8x\right) = 4$

$\left(y^2 - 4y + 4\right) - 4\left(x^2 + 2x\right) = 4 + 4$

$\left(y-2\right)^2 - 4\left(x+1\right)^2 = 4 + 4 - 4$

$\left(y-2\right)^2 - 4\left(x+1\right)^2 = 4$ Now divide both sides of the equation by 4.

$$\frac{\left(y-2\right)^2}{4} - \frac{4\left(x+1\right)^2}{4} = \frac{4}{4}$$

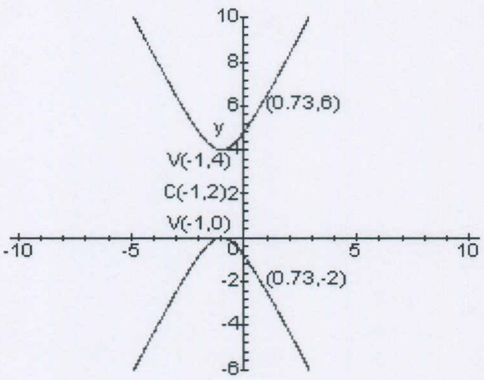
$\dfrac{\left(y-2\right)^2}{4} - \dfrac{\left(x+1\right)^2}{1} = 1$ This is the standard form of a hyperbola that opens up and down (since y-term is positive) with center $(-1, 2)$, vertices located 2 units above and below the center, $(-1, 4)$ and $(-1, 0)$. For two additional points let $x = \sqrt{3} - 1$ and solve for y.

$$\frac{\left(y-2\right)^2}{4} - \frac{\left(\left(\sqrt{3}-1\right)+1\right)^2}{1} = 1, \quad \frac{\left(y-2\right)^2}{4} - \frac{\left(\sqrt{3}\right)^2}{1} = 1, \quad \frac{\left(y-2\right)^2}{4} - \frac{3}{1} = 1,$$

$$\frac{\left(y-2\right)^2}{4} - 3 = 1, \quad \frac{\left(y-2\right)^2}{4} = 4, \quad \left(y-2\right)^2 = 16, \quad y - 2 = \pm 4.$$

When $y - 2 = 4$, $y = 6$ giving the point $(\sqrt{3} - 1,\ 6)$ which is approximately (0.73, 6).

When $y - 2 = -4$, $y = -2$ giving the point $(\sqrt{3} - 1,\ -2)$ which is approximately (0.73, -2).



Problem #49

$\boxed{1}$ $3x^2+2y^2=36$

$\boxed{2}$ $4x^2-y^2=4$ Begin by multiplying both sides of equation $\boxed{2}$ by 2.

$2\times\boxed{2}$: $8x^2-2y^2=8$ Now add equation $\boxed{1}$,

$\boxed{1}$: $\underline{3x^2+2y^2=36}$

$11x^2=44$

$x^2=4, \quad x=\pm 2$ Now substitute these values of x into equation $\boxed{2}$

When $x=2$, $\quad 4(2)^2-y^2=4, \quad 16-y^2=4, \quad y^2=12, \quad y=\pm\sqrt{12}=\pm 2\sqrt{3}$ giving the proposed solutions $\left(2,\ 2\sqrt{3}\right)$ and $\left(2,\ -2\sqrt{3}\right)$.

When $x=-2$, $\quad 4(-2)^2-y^2=4, \quad 16-y^2=4, \quad y^2=12, \quad y=\pm\sqrt{12}=\pm 2\sqrt{3}$ giving the proposed solutions $\left(-2,\ 2\sqrt{3}\right)$ and $\left(-2,\ -2\sqrt{3}\right)$.

All four proposed solutions check in both original equations so the solution set is $\left\{\left(2,\ 2\sqrt{3}\right),\ \left(2,\ -2\sqrt{3}\right),\left(-2,\ 2\sqrt{3}\right),\ \left(-2,\ -2\sqrt{3}\right)\right\}$

Problem #50

$\boxed{1}$ $\frac{1}{x}+\frac{3}{y}=4$

$\boxed{2}$ $\frac{2}{x}-\frac{1}{y}=7$

Let $u=\frac{1}{x}$, $v=\frac{1}{y}$, so the system becomes;

$\boxed{1}$ $u+3v=4$

$\boxed{2}$ $2u-v=7$ Add 3 times equation $\boxed{2}$ to equation $\boxed{1}$.

$$\begin{array}{ll} \boxed{1} & u+3v=4 \\ 3\times\boxed{2}: & \underline{6u-3v=21} \\ & 7u=25 \\ & u=\frac{25}{7} \end{array}$$

Now substitute this value for u into either original equation.

$\frac{25}{7}+3v=4$, $3v=4-\frac{25}{7}=\frac{3}{7}$, $v=\frac{1}{7}$. But we need x, y.

$u=\frac{25}{7}=\frac{1}{x}$, so $x=\frac{7}{25}$. $v=\frac{1}{7}=\frac{1}{y}$, so $y=7$.

The proposed solution $\left(\frac{7}{25}, 7\right)$ checks in both original equations so the solution set is $\left\{\left(\frac{7}{25}, 7\right)\right\}$.